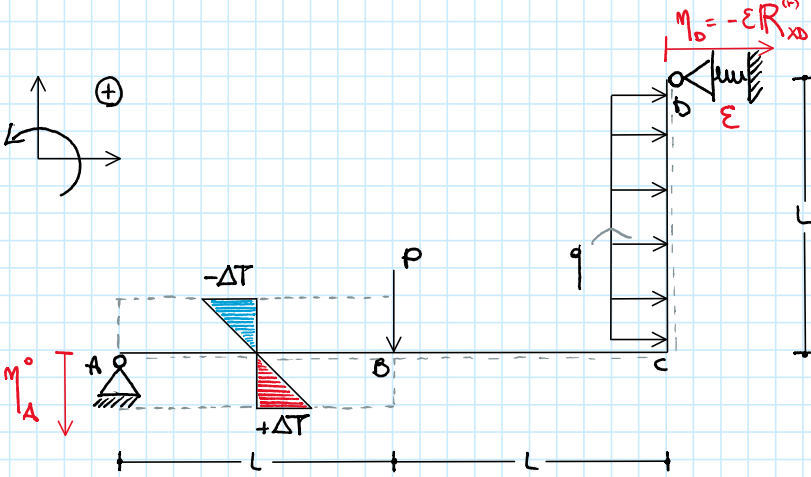


RISOLVERE LA STRUTTURA UNA VOLTA IPERSTATICA
DETERMINANDO IL DIAGRAMMA DEL MOMENTO



POSIZIONI

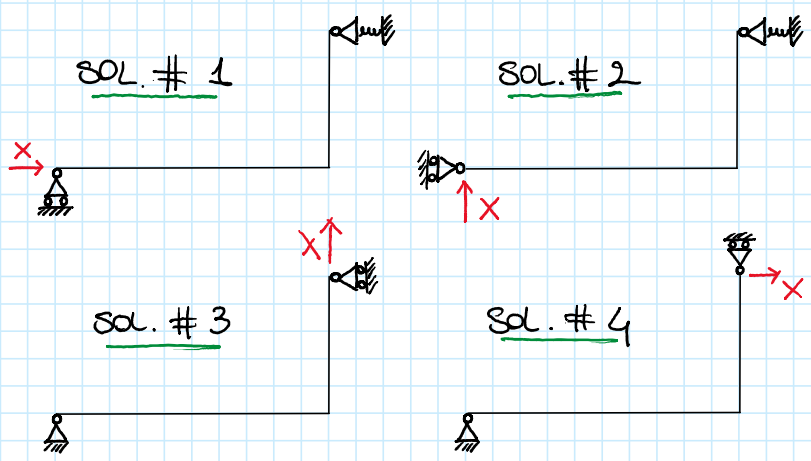
$$|P| = qL$$

$$\left| \frac{\alpha \Delta T}{h} \right| = \frac{qL^2}{EI}$$

$$|M_B^0| = \frac{qL^2}{EI}$$

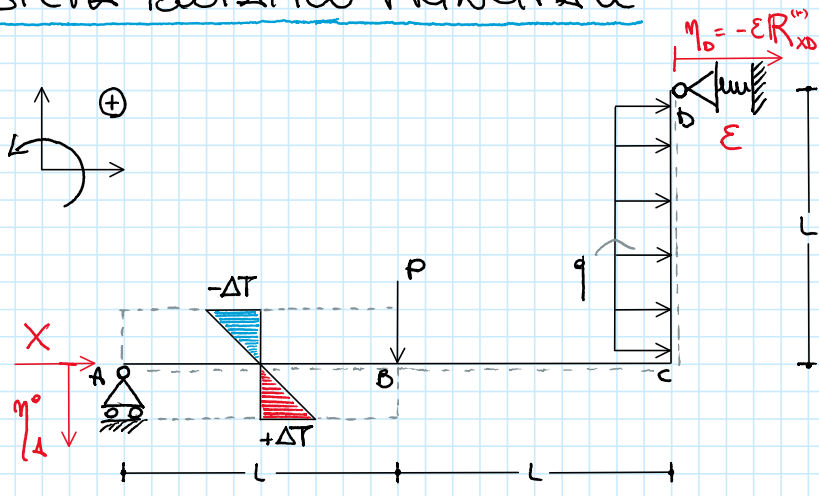
$$|\epsilon| = \frac{L^3}{EI}$$

SOLUZIONI POSSIBILI

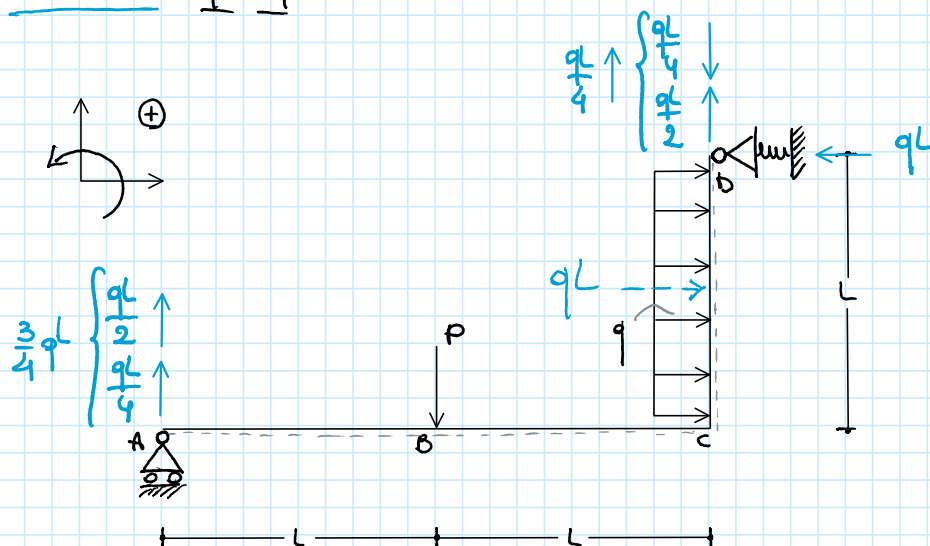


SOLUZIONE #1

• SISTEMA ISOSTATICO PRINCIPALE



SCHEMA [0] SOLO CARICHI ESTERNI



TRATTO AB $0 \leq z \leq L$

$\begin{matrix} A \\ \uparrow \frac{3}{4}qL \\ \rightarrow z \end{matrix} \Rightarrow \boxed{M^{(0)}(z) = \frac{3}{4}qLz} \quad \begin{cases} M_A = 0 \\ M_B = \frac{3}{4}qL^2 \end{cases}$

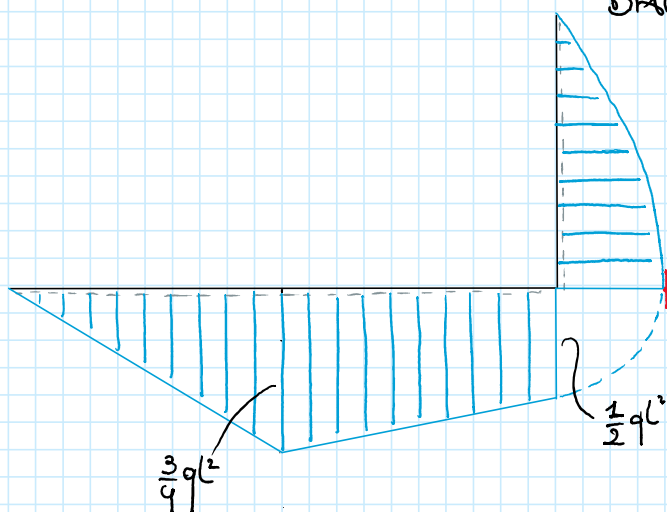
TRATTO BC $L \leq z \leq 2L$

$\begin{matrix} B \\ \downarrow qL \\ \uparrow \frac{3}{4}qL \\ \rightarrow z \end{matrix} \Rightarrow \boxed{M^{(0)} = \frac{3}{4}qL^2 + \frac{3}{4}qLz - qLz} = \boxed{-\frac{qL}{4}z + qL^2} \quad \begin{cases} M_B = \frac{3}{4}qL^2 \\ M_C = \frac{qL^2}{2} \end{cases}$

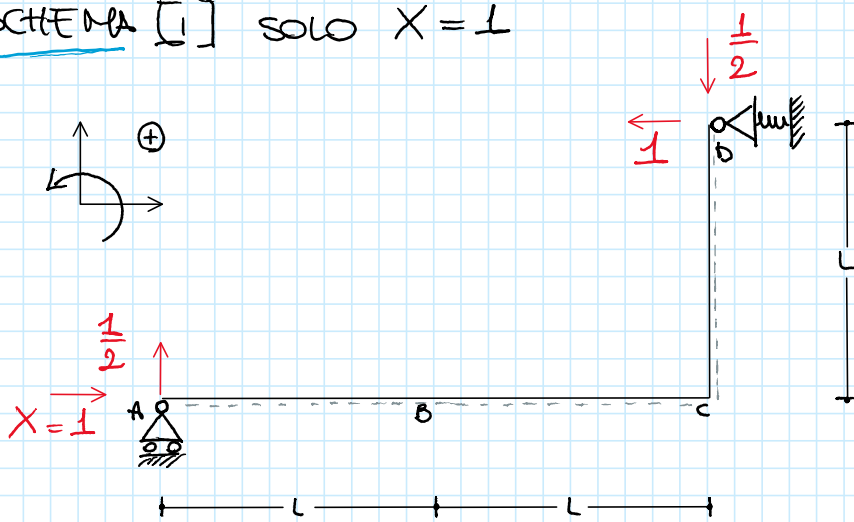
TRATTO CD $0 \leq z \leq L$

$\begin{matrix} \rightarrow z \\ \downarrow qL \end{matrix} \Rightarrow \boxed{M^{(0)}(z) = qL(L-z) - \frac{q(L-z)^2}{2}} = \boxed{\frac{qL^2}{2} - \frac{qz^2}{2}} \quad \begin{cases} M_C = \frac{qL^2}{2} \\ M_D = 0 \end{cases}$

DIAGRAMMA $M^{(0)}(z)$



SCHEMA [1] SOLO $X=1$



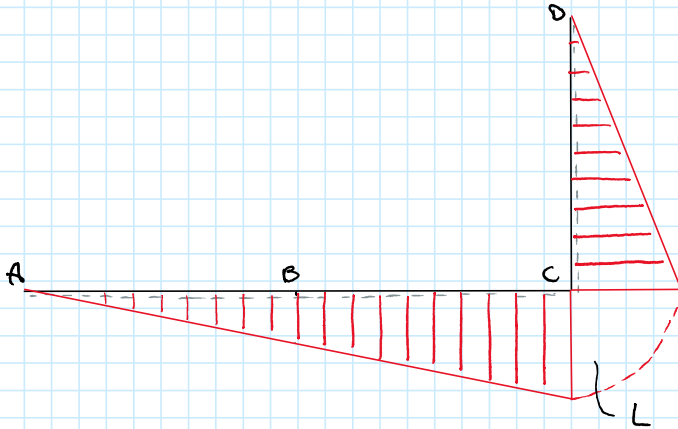
TRATTO AC $0 \leq z \leq 2L$

$$\begin{aligned} & \text{Diagram of segment AC: A horizontal beam of length } 2L \text{ with a horizontal force } 1 \text{ at A and a vertical force } \frac{1}{2} \text{ at C.} \\ & \Rightarrow \boxed{M''(z) = \frac{z}{2}} \quad \begin{cases} M_A = 0 \\ M_C = L \end{cases} \end{aligned}$$

TRATTO CD $0 \leq z \leq L$

$$\begin{aligned} & \text{Diagram of segment CD: A vertical beam of length } L \text{ with a horizontal force } \frac{1}{2} \text{ at C and a vertical force } 1 \text{ at D.} \\ & \Rightarrow \boxed{M'''(z) = L - z} \quad \begin{cases} M_C = L \\ M_D = 0 \end{cases} \end{aligned}$$

DIGRAMMA $M''(z)$



$$\underline{\alpha_{ve}} = 1 \cdot \eta_i^{(n)} + \sum_j R_j^{(f)} \cdot \eta_j^{(n)} =$$

$$\begin{aligned} &= \underbrace{R_{y_A}^{(n)} \cdot \eta_A^0}_{-\frac{\eta_A^0}{2}} + \underbrace{R_{x_D}^{(n)} \cdot \eta_D^{(n)}}_{-1 \cdot \underbrace{\varepsilon(R_{x_D}^{(n)})}_{R_{x_D}^{(n)} + R_{x_D}^{(n)} X}} \\ &= -\frac{\eta_A^0}{2} - \varepsilon[qL + X] \end{aligned}$$

$$\alpha_{Vi} = \int_{Str} M^{(e)} \frac{M^{(m)}}{EI} + \int_{Str} M^{(e)} \frac{\alpha \Delta T}{h} dStr =$$

$\hookrightarrow M^{(m)} = M^{(0)} + M^{(1)} X$

$$= \int_{Str} M^{(m)} \frac{M^{(m)}}{EI} dStr + X \int_{Str} \frac{[M^{(m)}]^2}{EI} dStr + \int_{Str} M^{(m)} \frac{\alpha \Delta T}{h} dStr =$$

$$= \frac{1}{EI} \left\{ \int_{AB} \frac{7}{2} \left[\frac{3}{4} qLz^2 \right] dz + \int_{BC} \frac{7}{2} \left[-\frac{qL}{4} z + qL^2 \right] dz + \int_{CD} (1-z) \left[\frac{qL^2}{2} - \frac{q}{2} z^2 + \frac{q}{2} z^3 \right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_{AB} \frac{z^2}{4} dz + \int_{CD} (L-z)^2 dz \right\} + \frac{\alpha \Delta T}{h} \int_{AB} \frac{z}{2} dz =$$

$$= \frac{1}{EI} \left\{ \int_0^L \frac{3}{8} qLz^2 dz + \int_L^{2L} \left[-\frac{qL}{8} z^2 + \frac{qL^2}{2} z \right] dz + \int_0^L \left[\frac{qL^3}{2} - \frac{qL}{2} z^2 - \frac{qL^2}{2} z + \frac{q}{2} z^3 \right] dz \right\} +$$

$$+ \frac{X}{EI} \left\{ \int_0^{2L} \frac{z^2}{4} dz + \int_0^L [L^2 + z^2 - 2Lz] dz \right\} + \frac{\alpha \Delta T}{h} \int_0^L \frac{z}{2} dz =$$

$$= \frac{1}{EI} \left\{ \frac{3}{8} qL \left[\frac{z^3}{3} \right]_0^L - \frac{qL}{8} \left[\frac{z^3}{3} \right]_L^{2L} + \frac{qL^2}{2} \left[\frac{z^2}{2} \right]_L^{2L} + \frac{qL^3}{2} \left[z \right]_0^L - \frac{qL}{2} \left[\frac{z^3}{3} \right]_0^L - \right.$$

$$\left. - \frac{qL^2}{2} \left[\frac{z^2}{2} \right]_0^L + \frac{q}{2} \left[\frac{z^4}{4} \right]_0^L \right\} + \frac{X}{EI} \left\{ \frac{1}{4} \left[\frac{z^3}{3} \right]_0^{2L} + L^2 \left[z \right]_0^L + \left[\frac{z^3}{3} \right]_0^L - 2L \left[\frac{z^2}{2} \right]_0^L \right\} +$$

$$+ \frac{\alpha \Delta T}{2h} \left[\frac{z^2}{2} \right]_0^L =$$

$$= \frac{qL^4}{EI} \left[\frac{1}{8} - \frac{7}{24} + \frac{3}{4} + \frac{1}{2} - \frac{1}{6} - \frac{1}{4} + \frac{1}{8} \right] + \frac{XL^3}{EI} \left[\frac{4}{12} + \frac{1}{3} \right] + \frac{\alpha \Delta T L^2}{4h} =$$

$$= \frac{19}{24} \frac{qL^4}{EI} + \frac{XL^3}{EI} + \frac{\alpha \Delta T L^2}{4h}$$

$\Delta u = \Delta u_i$ fornisce

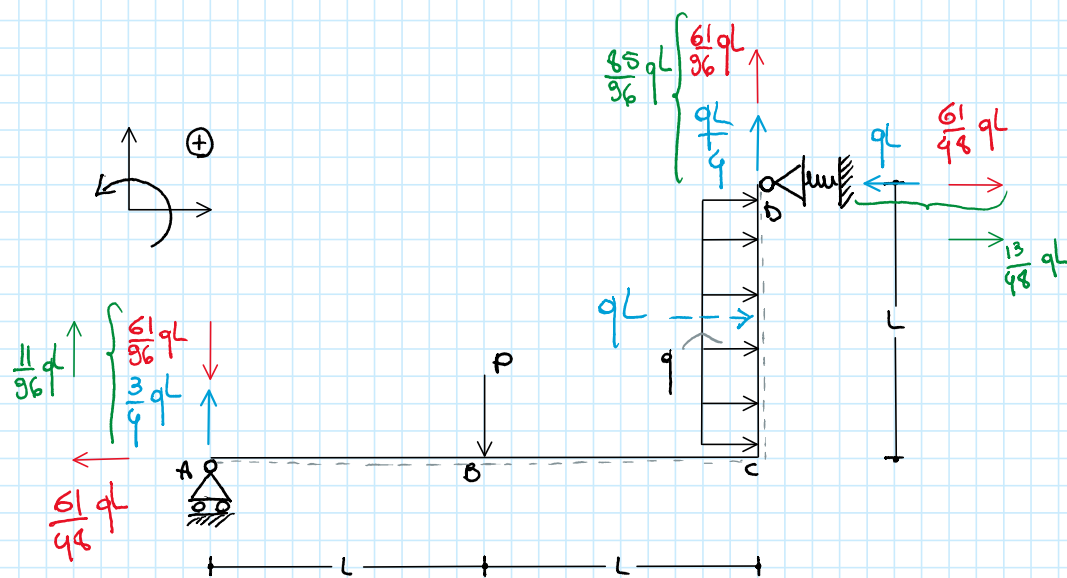
$$-\frac{\eta^0}{2} - \varepsilon qL - \varepsilon X = \frac{19}{24} \frac{qL^4}{EI} + \frac{X L^3}{EI} + \frac{\alpha \Delta T L^2}{4h}$$

$$X \left[\frac{L^3}{EI} + \varepsilon \right] = - \left\{ \frac{\eta^0}{2} + \varepsilon qL + \frac{19}{24} \frac{qL^4}{EI} + \frac{\alpha \Delta T L^2}{4h} \right\}$$

$$X \left[\frac{L^3}{EI} + \frac{L^3}{EI} \right] = - \left[\frac{qL^4}{2EI} + \frac{qL^4}{EI} + \frac{19}{24} \frac{qL^4}{EI} + \frac{qL^4}{4EI} \right]$$

$$X = -\frac{61}{48} qL \quad \text{Negativo, verso contrario}$$

SOLUZIONE SISTEMA ISOSTATICO PRINCIPALE



TRATTO AB $0 \leq z \leq L$

Free body diagram of segment AB shows a horizontal reaction of $\frac{61}{96}qL$ to the left at A and a vertical reaction of $\frac{11}{96}qL$ upwards at A. The bending moment equation is:

$$M''(z) = \frac{11}{96}qL^2$$

Boundary conditions:

$$\begin{cases} M_A = 0 \\ M_B = \frac{11}{96}qL^2 \end{cases}$$

TRATTO BC $L \leq z \leq 2L$

Free body diagram of segment BC shows a vertical reaction of $\frac{11}{96}qL$ upwards at B and a horizontal reaction of $\frac{13}{48}qL$ to the right at C. The bending moment equation is:

$$M''(z) = \frac{11}{96}qL^2 - q(z-L)$$

Boundary conditions:

$$\begin{cases} M_B = \frac{11}{96}qL^2 \\ M_C = -\frac{37}{48}qL^2 \end{cases}$$

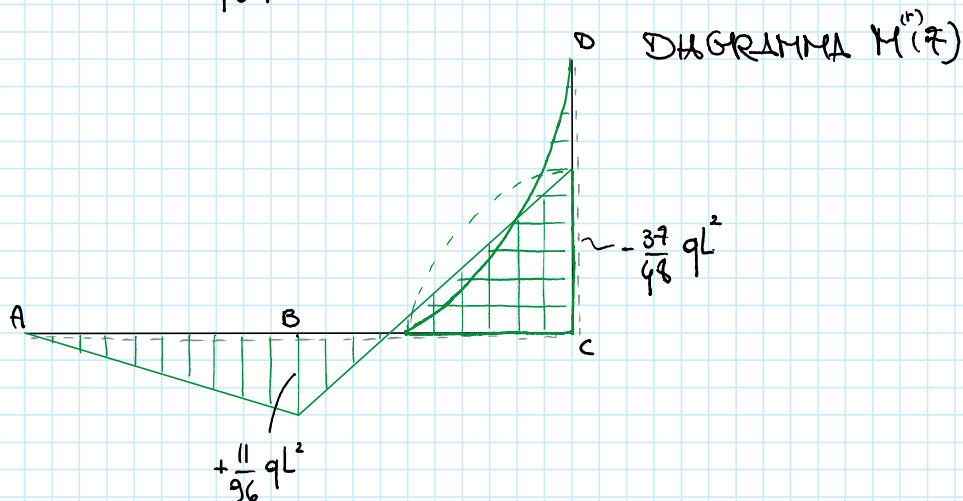
TRATTO CD $0 \leq z \leq L$

Free body diagram of segment CD shows a uniformly distributed load q acting downwards over a length L . The bending moment equation is:

$$M''(z) = -\frac{13}{48}qL(L-z) - \frac{q(L-z)^2}{2}$$

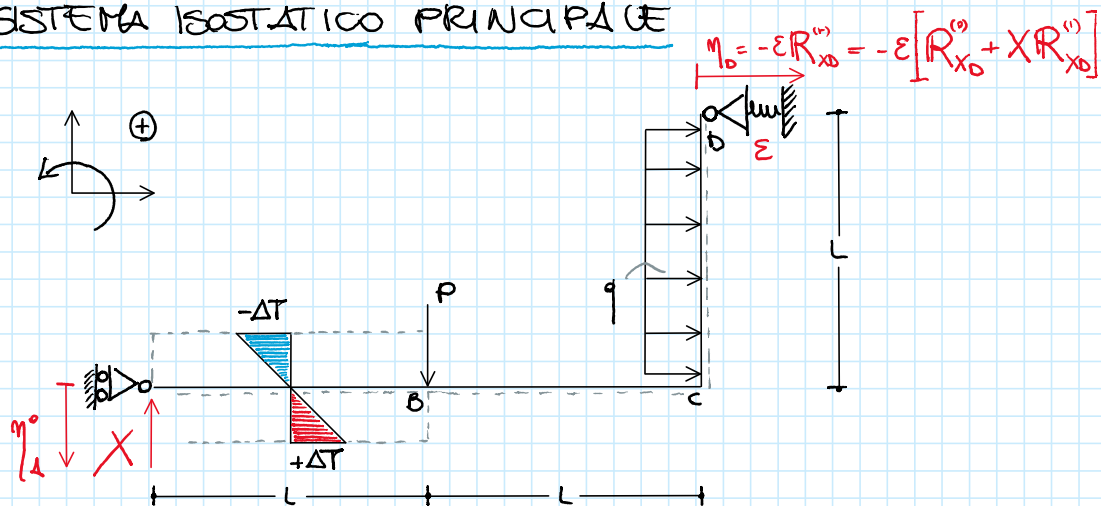
Boundary conditions:

$$\begin{cases} M_C = -\frac{37}{48}qL^2 \\ M_D = 0 \end{cases}$$

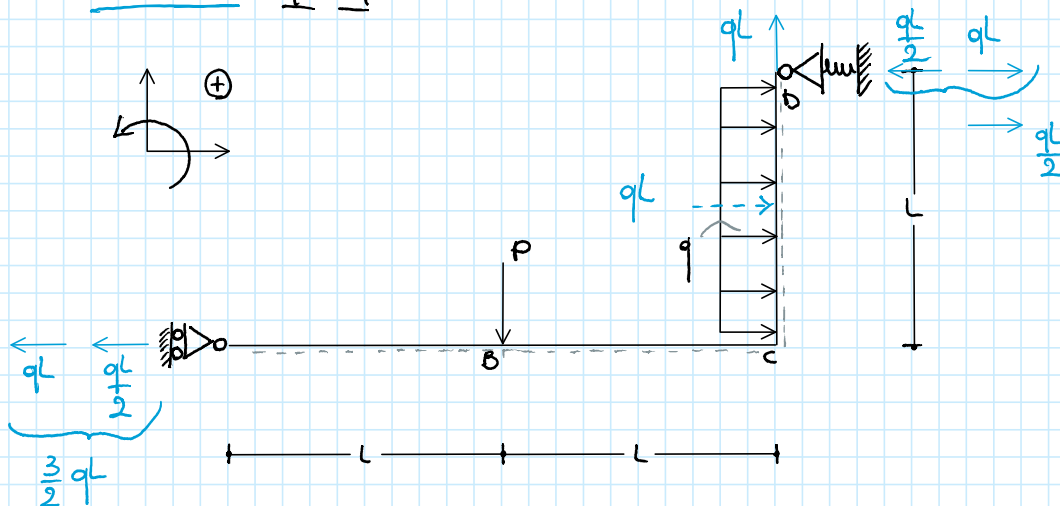


SOLUZIONE # 2

SISTEMA ISOSTATICO PRINCIPALE



SCHEMA [0] SOLO CARICHI ESTERNI



TRATTO AB $0 < z \leq L$

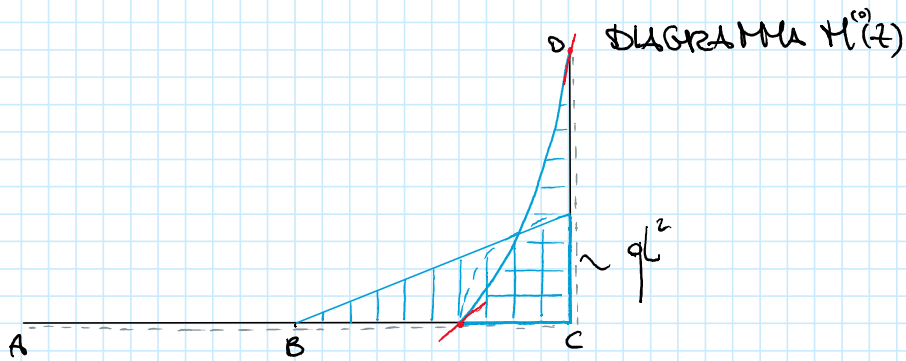
$$M^{(0)}(z) = \emptyset$$

TRATTO BC $L \leq z \leq 2L$

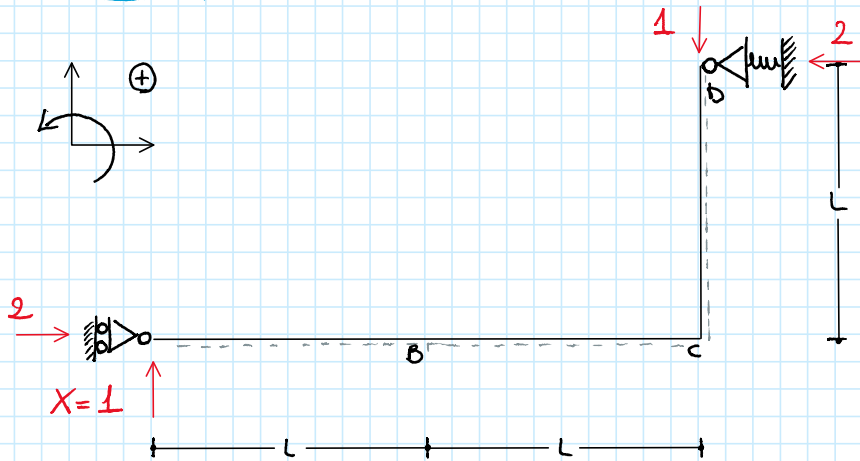
$$M^{(0)}(z) = -qL(z-L) \quad \begin{cases} M_B = 0 \\ M_C = -qL^2 \end{cases}$$

TRATTO CD

$$M^{(0)}(z) = -\frac{qL}{2}(L-z) - \frac{q(L-z)^2}{2} \quad \begin{cases} M_C = -qL^2 \\ M_D = \emptyset \end{cases}$$



SCHEMA [1] SOLO $X=1$



TRATTO AB $0 \leq z \leq L$

$\begin{matrix} \text{A} \\ \text{---} \\ \text{B} \end{matrix} \quad \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} \quad \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix}$
 $\Rightarrow M''(z) = z \quad \begin{cases} M_A = 0 \\ M_B = L \end{cases}$

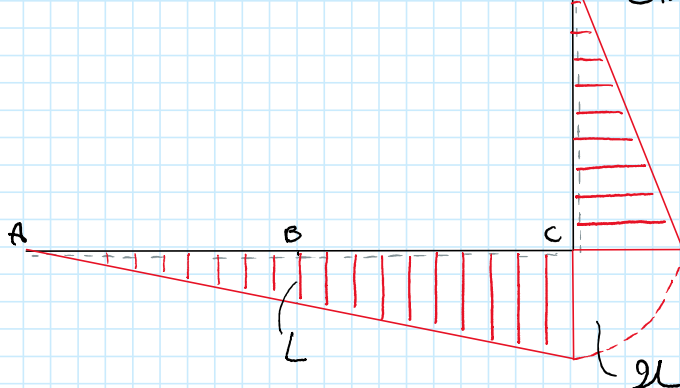
TRATTO BC $L \leq z \leq 2L$

Come tratto BC $M''(z) = z$
 $\begin{cases} M_B = L \\ M_C = 2L \end{cases}$

TRATTO CD $0 \leq z \leq L$

$\begin{matrix} \text{D} \\ \text{---} \\ \text{C} \end{matrix} \quad \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} \quad \begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix}$
 $\Rightarrow M''(z) = 2(L-z)$
 $\begin{cases} M_C = 2L \\ M_D = 0 \end{cases}$

DIAGRAMMA $M''(z)$



$$\begin{aligned} \underline{\mathcal{L}_{ve}} &= 1 \cdot (-\eta_A^0) + \sum_j R_j^{(v)} \eta_j^{(v)} = -\eta_A^0 + \underbrace{R_{x_0}^{(v)}}_{-2} \cdot \underbrace{\eta_0^{(v)}}_{\frac{qL}{2}} = \\ &= -\eta_A^0 + 2\varepsilon \left[\frac{qL}{2} - 2X \right] \end{aligned}$$

$-\varepsilon \{ \underbrace{R_{x_0}^{(v)}}_{\frac{qL}{2}} + \underbrace{R_{x_0}^{(v)} X}_{-2} \}$

$$\begin{aligned} \underline{\mathcal{L}_{vi}} &= \int_{Str} M'' M'' dStr + \int_{Str} M'' \frac{\alpha \Delta T}{h} dStr = \\ &= \frac{1}{EI} \left\{ \int_{Str} M'' M'' dStr \right\} + \frac{X}{EI} \left\{ \int_{Str} [M'']^2 dStr \right\} + \int_{Str} M'' \frac{\alpha \Delta T}{h} dStr = \\ &= \frac{1}{EI} \left\{ \int_{BC} M'' M'' dz + \int_{CD} M'' M'' dz \right\} + \frac{X}{EI} \left\{ \int_{AC} [M'']^2 dz + \int_{CD} [M'']^2 dz \right\} + \frac{\alpha \Delta T}{h} \int_{AB} M'' dz = \\ &= \frac{1}{EI} \left\{ \int_0^{2L} \underbrace{z[-qL(z-L)]}_{-qL(z^2-Lz)} dz + \int_0^L \underbrace{2(L-z) \left[-\frac{qL}{2}(L-z) - \frac{q(L-z)^2}{2} \right]}_{(L-z)[-qL(L-z) - q(L^2+z^2-2Lz)]} dz \right\} + \\ &+ \frac{X}{EI} \left\{ \int_0^{2L} z^2 dz + \int_0^L \underbrace{4(L-z)^2}_{L^2+z^2-2Lz} dz \right\} + \frac{\alpha \Delta T}{h} \int_0^L z dz = \\ &= \frac{1}{EI} \left\{ -qL \left[\frac{z^3}{3} \right]_0^{2L} + qL^2 \left[\frac{z^2}{2} \right]_0^{2L} - 2qL^3 \left[z \right]_0^L - qL \left[\frac{z^3}{3} \right]_0^L + 3qL \left[\frac{z^2}{2} \right]_0^L + \right. \\ &+ 2qL^2 \left[\frac{z^2}{2} \right]_0^L + q \left[\frac{z^4}{4} \right]_0^L - 3qL \left[\frac{z^3}{3} \right]_0^L \left. \right\} + \frac{X}{EI} \left\{ \left[\frac{z^3}{3} \right]_0^{2L} + 4L^2 \left[z \right]_0^L + \right. \\ &+ 4 \left[\frac{z^3}{3} \right]_0^L - 8L \left[\frac{z^2}{2} \right]_0^L \left. \right\} + \frac{\alpha \Delta T}{h} \left[\frac{z^2}{2} \right]_0^L = \\ &= \frac{1}{EI} \left\{ -\frac{7}{3} qL^4 + \frac{3}{2} qL^4 - 2qL^4 - qL \frac{L^3}{3} + 3qL^2 \frac{L^2}{2} + 2qL^2 \frac{L^2}{2} + \frac{qL^4}{4} - 3qL \frac{L^3}{3} \right\} + \\ &+ \frac{X}{EI} \left\{ \frac{1}{3} 8L^3 + 4L^2 \cdot L + 4 \frac{L^3}{3} - 8L \frac{L^2}{2} \right\} + \frac{\alpha \Delta T}{h} \frac{L^2}{2} \\ &= \underline{-\frac{17}{12} \frac{qL^4}{EI} + \frac{4L^3}{EI} X + \frac{\alpha \Delta T}{h} \frac{L^2}{2}} \end{aligned}$$

$L_{re} = L_{vi}$ fornisce

$$-\eta_1^0 + 2\varepsilon \left[\frac{qL}{2} - 2X \right] = -\frac{5}{12} \frac{qL^4}{EI} + \frac{4L^3}{EI} X + \frac{\alpha \Delta T}{h} \frac{L^2}{2}$$

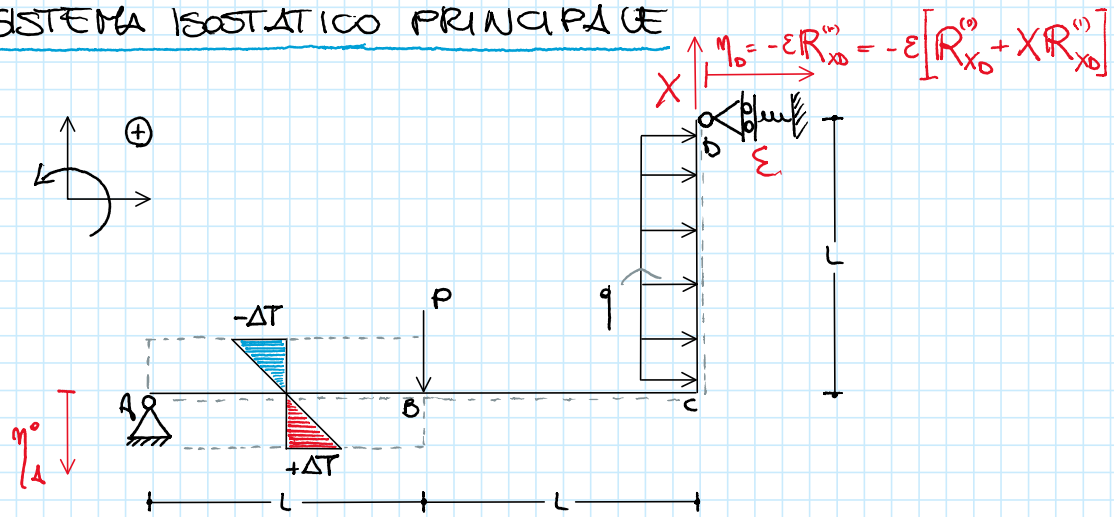
$$-\frac{qL^4}{EI} + \frac{qL^4}{EI} - \frac{4L^3}{EI} X = -\frac{17}{12} \frac{qL^4}{EI} + \frac{4L^3}{EI} X + \frac{qL^4}{2EI}$$

$$-4X = qL \left(\frac{1}{2} - \frac{17}{12} \right)$$

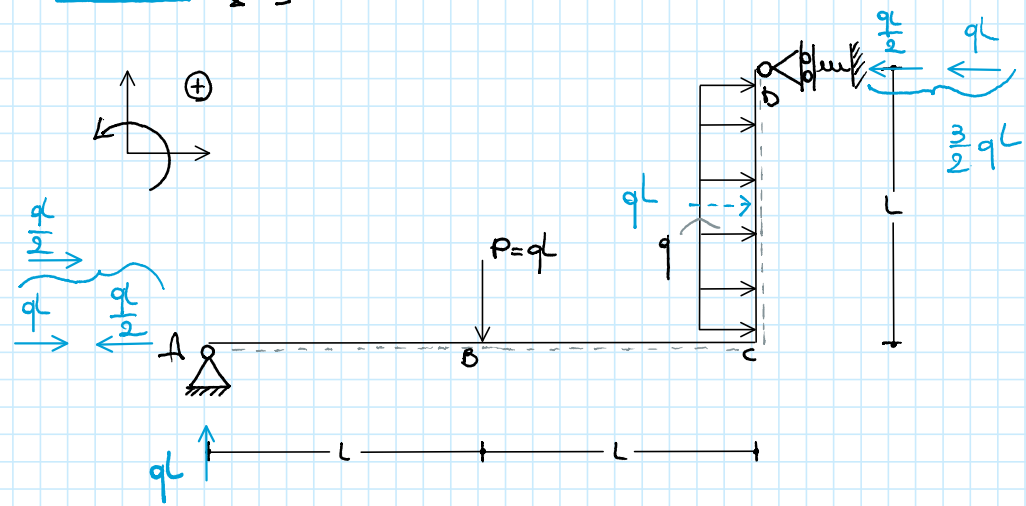
$$X = + \frac{11}{96} qL \quad \text{POSITIVO}$$

SOLUZIONE # 3

SISTEMA ISOSTATICO PRINCIPALE



SCHEMA [0] SOLO CARICHI ESTERNI



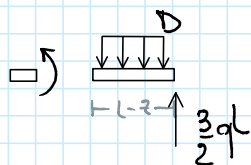
TRATTO AB $0 < z \leq L$

$\frac{qL}{2} \rightarrow$ \leftarrow $M^{(0)}(z) = qLz$ $\begin{cases} M_A = 0 \\ M_B = qL^2 \end{cases}$

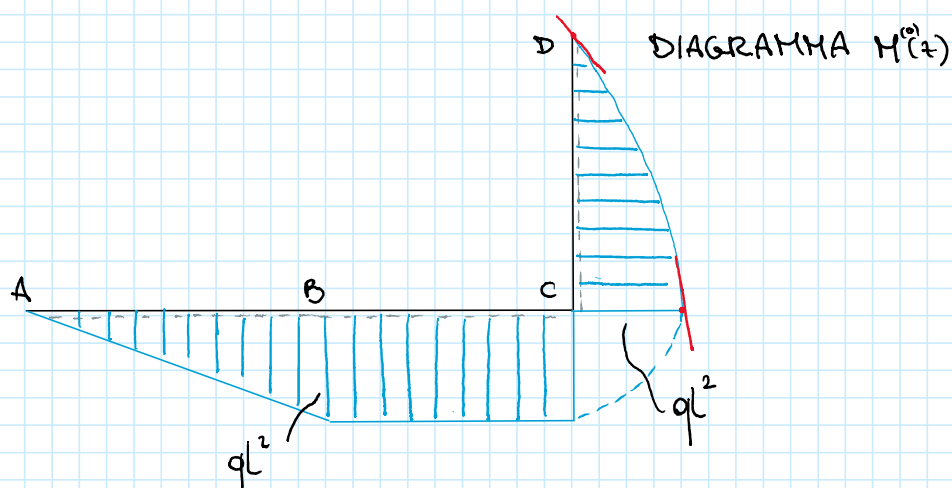
TRATTO BC $0 \leq z \leq L$

$\frac{qL}{2} \rightarrow$ \leftarrow $M^{(0)}(z) = qL^2$ cost.

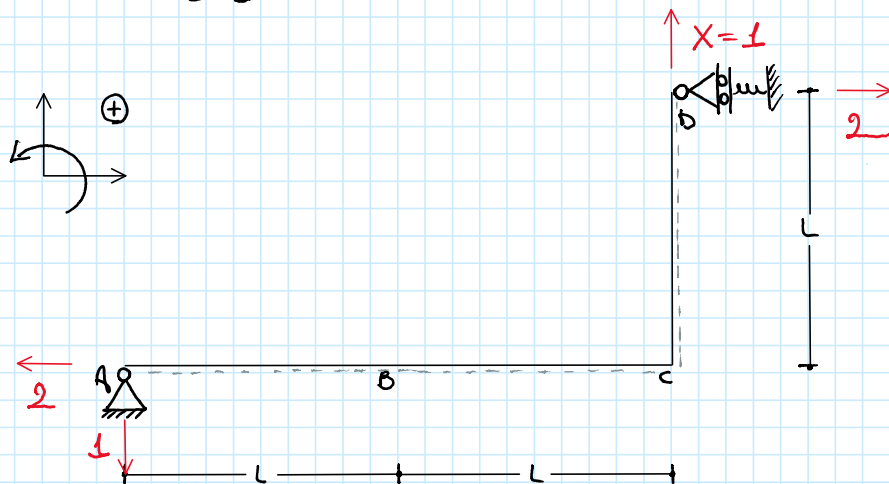
TRATTO CD $0 \leq z \leq L$



$$\begin{aligned}
 M^{(0)}(z) &= \frac{3}{2} qL(L-z) - \frac{q}{2}(L-z)^2 \\
 &= \frac{3}{2} qL^2 - \frac{3}{2} qLz - \frac{1}{2} qL^2 + \frac{1}{2} qz^2 + qLz \\
 &= \boxed{qL^2 - \frac{1}{2} qz^2 - \frac{1}{2} qLz}
 \end{aligned}
 \quad \begin{cases} M_c = qL^2 \\ M_b = 0 \end{cases}$$



SCHEMA [1] SOLO $X=1$



TRATTO AB $0 \leq z \leq L$

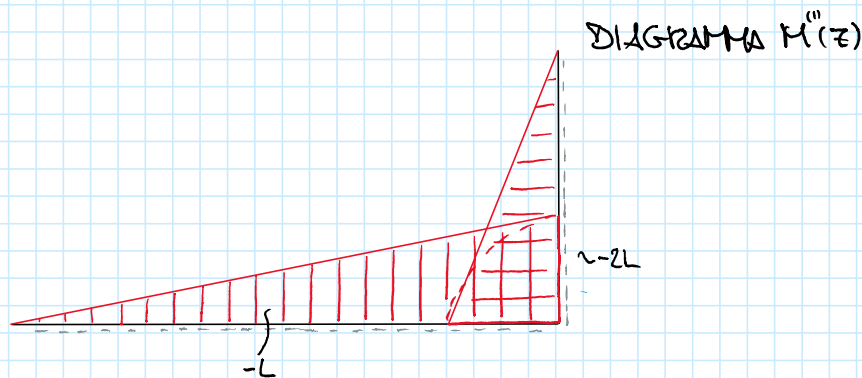
$$\begin{aligned}
 M^{(0)}(z) &= -z
 \end{aligned}
 \quad \begin{cases} M_A = 0 \\ M_B = -L \end{cases}$$

TRATTO BC $0 \leq z \leq L$

$$\begin{aligned}
 M^{(0)}(z) &= -L - z
 \end{aligned}
 \quad \begin{cases} M_B = -L \\ M_C = -2L \end{cases}$$

TRATTO CD $0 \leq z \leq L$

$$\Rightarrow \begin{array}{c} D \\ \xrightarrow{L} \\ \downarrow 2 \\ L-z \end{array} \quad M''(z) = -2(L-z) \quad \begin{cases} M_B = -2L \\ M_C = 0 \end{cases}$$



$$\begin{aligned} \alpha_{ve} &= 1 \cdot \eta_i^{(r)} + \sum_j R_j^{(r)} \cdot \eta_j^{(r)} = \\ &= \underbrace{R_{yA}^{(r)}}_{-1} \cdot \underbrace{\eta_A^0}_{-\eta_A^0} + \underbrace{R_{xD}^{(r)}}_{+2} \cdot \underbrace{\eta_D^{(r)}}_{-\varepsilon R_{xD}^{(r)}} = \\ &= + \eta_A^0 - 2\varepsilon \left[-\frac{3}{2}qL + 2X \right] \underbrace{(R_{xD}^{(r)} + R_{xD}^{(r)}X)}_{-\frac{3}{2}qL + 2X} \end{aligned}$$

$$\begin{aligned} \alpha_{vi} &= \int_{Str} M^{(r)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(r)} \frac{\alpha \Delta \bar{T}}{h} dStr = \\ &= \int_{Str} M^{(r)} \frac{M^{(r)}}{EI} dStr + X \int_{Str} \frac{[M^{(r)}]^2}{EI} dStr + \int_{Str} M^{(r)} \frac{\alpha \Delta \bar{T}}{h} dStr = \\ &= \frac{1}{EI} \left\{ \int_{AB} (-t)(qL) dt + \int_{BC} \underbrace{-(L+t)qL^2}_{-qL^3 - qL^2 t} dt + \int_{CD} \underbrace{-2(L-t)(qL^2 - \frac{1}{2}qL^2 t - \frac{1}{2}qL^2 t)}_{-2qL^3 + qL^2 t + qL^2 t - 2qL^2 t - qL^2 t - qL^2 t} dt \right\} + \\ &+ \frac{X}{EI} \left\{ \int_{AB} (-t)^2 dt + \int_{BC} \underbrace{[-(L+t)]^2}_{qL^2 + 4qL^2 t - 4qL^2 t} dt + \int_{CD} \underbrace{[-2(L-t)]^2}_{4L^2 - 4qL^2 t + 4qL^2 t} dt \right\} + \frac{\alpha \Delta \bar{T}}{h} \int_{AB} (-t) dt = \\ &= \frac{1}{EI} \left\{ -qL \left[\frac{t^3}{3} \right]_0^L - qL^3 \left[t \right]_0^L - qL^2 \left[\frac{t^2}{2} \right]_0^L - 2qL^3 \left[t \right]_0^L + qL \left[\frac{t^3}{3} \right]_0^L + qL^2 \left[\frac{t^2}{2} \right]_0^L + \right. \end{aligned}$$

$$\begin{aligned}
& + 2qL^2 \left[\frac{z^2}{2} \right]_0^L - q \left[\frac{z^4}{4} \right]_0^L - qL \left[\frac{z^3}{3} \right]_0^L \left\} + \frac{X}{EI} \left\{ \left[\frac{z^3}{3} \right]_0^L + L^2 \left[z \right]_0^L + \left[\frac{z^3}{3} \right]_0^L + 2L \left[\frac{z^2}{2} \right]_0^L \right. \\
& \left. + 4L^2 \left[z \right]_0^L + 4 \left[\frac{z^3}{3} \right]_0^L - 4L \left[\frac{z^2}{2} \right]_0^L \right\} - \frac{\alpha \Delta T}{h} \left[\frac{z^2}{2} \right]_0^L = \\
& = \frac{1}{EI} \left\{ -\frac{qL^4}{3} - qL^4 - \frac{qL^4}{2} - 2qL^4 + \frac{qL^4}{3} + \frac{qL^4}{2} + qL^4 - \frac{qL^4}{4} - \frac{qL^4}{3} \right\} + \\
& + \frac{X}{EI} \left\{ \frac{L^3}{3} + L^3 + \frac{L^3}{3} + L^3 + 4L^3 + \frac{4}{3}L^3 - 4L^3 \right\} - \frac{\alpha \Delta T}{h} \cdot \frac{L^2}{2} = \\
& = -\frac{31}{12} \frac{qL^4}{EI} + 4 \frac{XL^3}{EI} - \frac{L^2}{2} \frac{\alpha \Delta T}{h}
\end{aligned}$$

$\Delta_{ue} = \Delta_{vi}$ fornisce

$$+ \eta_A^0 - 2E \left[-\frac{3}{2} qL + 2X \right] = -\frac{31}{12} \frac{qL^4}{EI} + 4 \frac{XL^3}{EI} - \frac{L^2}{2} \frac{\alpha \Delta T}{h}$$

$$+ \frac{qL^4}{EI} + \frac{L^3}{EI} 3qL - \frac{XL^3}{EI} 4 = -\frac{31}{12} \frac{qL^4}{EI} + 4 \frac{XL^3}{EI} - \frac{1}{2} \frac{qL^4}{EI}$$

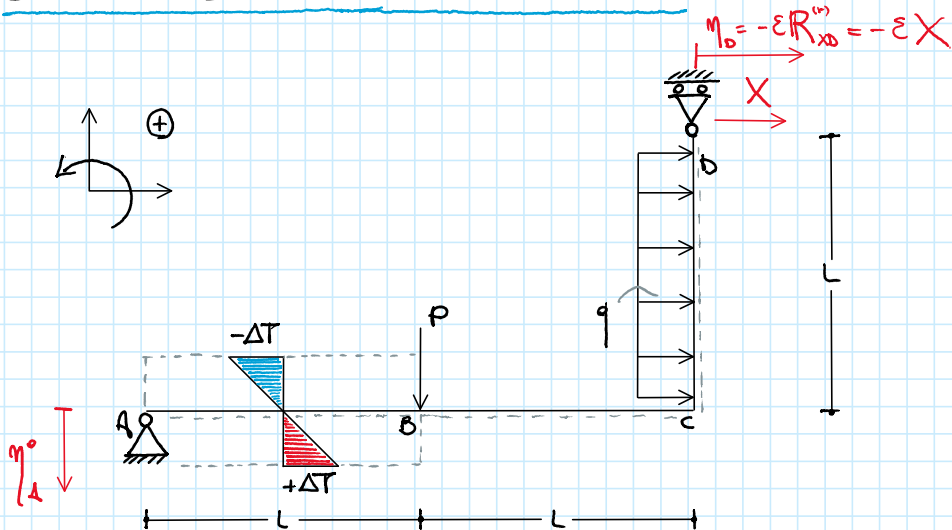
$$+ 8 \frac{XL^3}{EI} = \frac{qL^4}{EI} \left[+1 + 3 + \frac{31}{12} + \frac{1}{2} \right]$$

$$+ 8 \frac{XL^3}{EI} = \frac{qL^4}{EI} \left[\frac{85}{12} \right]$$

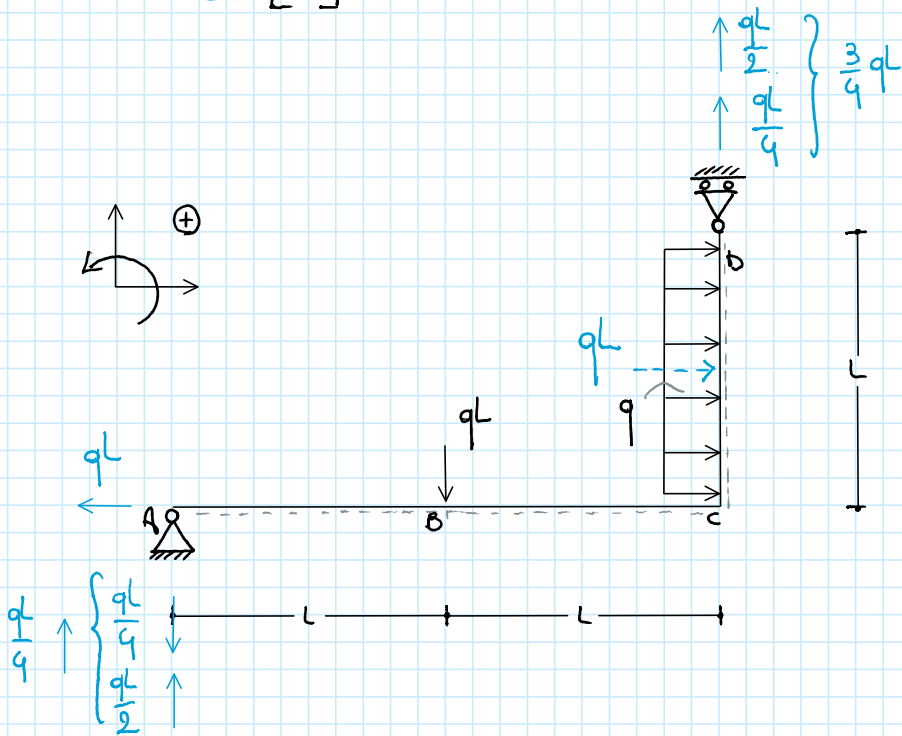
$$X = \frac{85}{96} qL \quad \text{Positivo, verso ok!}$$

SOLUZIONE # 4

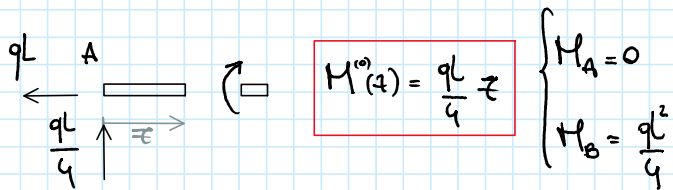
- SISTEMA ISOSTATICO PRINCIPALE



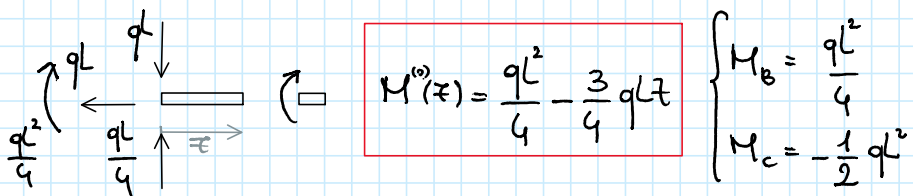
SCHEMA [0] SOLO CARICHI ESTERNI



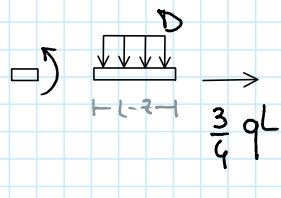
TRATTO AB $0 \leq t \leq L$



TRATTO BC $0 \leq z \leq L$

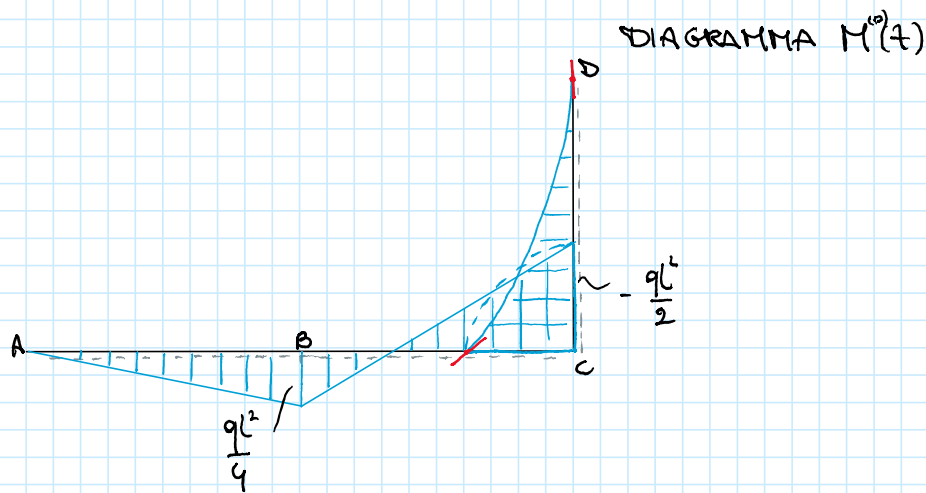


TRATTO CD $0 \leq z \leq L$

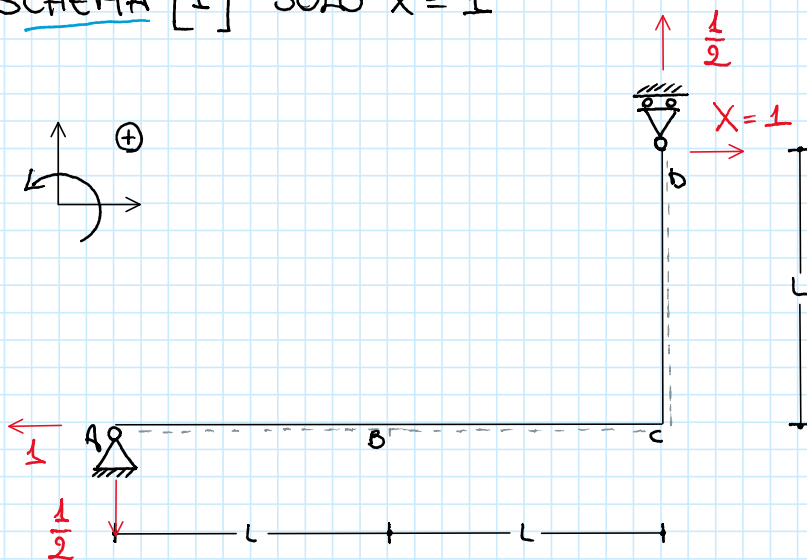


$$M''(z) = -\frac{q}{2}(L-z)^2$$

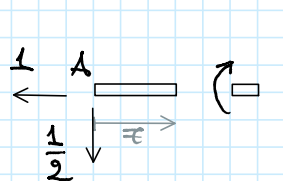
$$\begin{cases} M_C = -\frac{1}{2}qL^2 \\ M_D = 0 \end{cases}$$



SCHEMA [1] SOLO $X=1$



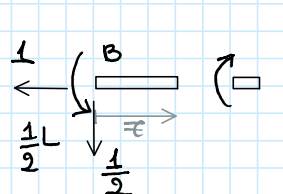
TRATTO AB $0 \leq z \leq L$



$$M'(z) = -\frac{1}{2}z$$

$$\begin{cases} M_A = 0 \\ M_B = -\frac{1}{2}L \end{cases}$$

TRATTO BC $0 \leq z \leq L$



$$M'(z) = -\frac{1}{2}(L+z)$$

$$\begin{cases} M_B = -\frac{1}{2}L \\ M_C = -L \end{cases}$$

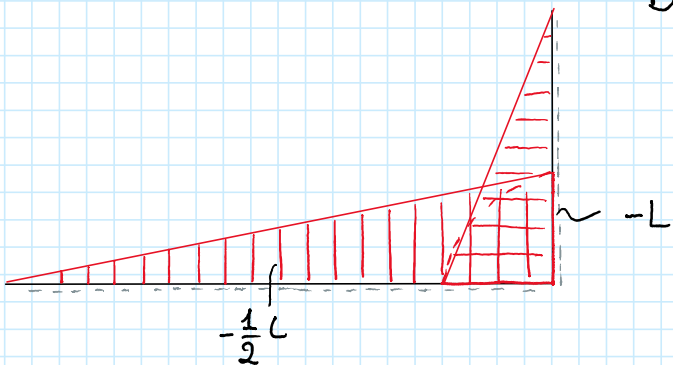
Tratto CD $0 \leq i \leq L$

Diagram of a horizontal bar of length L with a unit load 1 applied at distance $L-z$ from the left end. The distance from the right end to the load is z . The load is represented by a downward arrow labeled 1 . The distance from the left end to the load is $L-z$. The distance from the right end to the load is z .

$$M(z) = -(L-z)$$

$$\begin{cases} M_c = -L \\ M_D = 0 \end{cases}$$

DIAGRAMMA $M^{(n)}(7)$



$$\begin{aligned} \underline{L_{ve}} &= 1 \cdot \eta_i^{(1)} + \sum_j R_j^{(1)} \cdot \eta_j^{(1)} = \\ &= 1 \cdot \eta_b^{(1)} + R_{y_A}^{(1)} \cdot \eta_A^{(1)} \\ &\quad \quad \quad \underline{-\varepsilon X} \quad \quad \underline{-\frac{1}{2}} \quad \underline{-\eta_A^{(1)}} \\ &= \underline{-\varepsilon X + \frac{1}{2} \eta_A^{(1)}} \end{aligned}$$

$$\underline{L_{vi}} = \int_{Str} M^{(f)} \frac{M^{(r)}}{EI} dStr + \int_{Str} M^{(f)} \frac{\alpha \Delta T}{h} dStr =$$

$$\begin{aligned}
 &= \int_{Str} M^{(1)} \frac{M^{(0)}}{EI} dStr + X \int_{Str} \left[\frac{M^{(1)}}{EI} \right]^2 dStr + \int_{Str} M^{(1)} \frac{\alpha \Delta T}{h} dStr = \\
 &= \frac{1}{EI} \left\{ \int_{AB} -\frac{1}{2} z \left[\frac{qL}{4} z \right] dz + \int_{BC} -\frac{1}{2} (L+z) \left[\frac{qL^2}{4} - \frac{3}{4} qLz \right] dz + \int_{CD} \frac{q}{2} (L-z)^2 \frac{q}{2} (L-z)^2 dz \right\} + \\
 &+ \frac{X}{EI} \left\{ \int_{AB} \left(-\frac{1}{2} z \right)^2 dz + \int_{BC} \left[-\frac{1}{2} (L+z) \right]^2 dz + \int_{CD} \left[-(L-z) \right]^2 dz + \frac{\alpha \Delta T}{h} \int_{AB} -\frac{1}{2} z dz \right. \\
 &\quad \left. + \frac{L^2}{4} + \frac{7}{4} + \frac{7L}{2} \right\} + \frac{1}{EI} \left\{ -\frac{q}{8} \left[\frac{z^3}{3} \right]_0^L - \frac{qL^3}{8} \left[z \right]_0^L - \frac{qL^2}{8} \left[\frac{z^2}{2} \right]_0^L + \frac{3}{8} qL^2 \left[\frac{z^2}{2} \right]_0^L + \frac{3}{8} qL \left[\frac{z^3}{3} \right]_0^L + \frac{qL^3}{2} \left[z \right]_0^L - \frac{q}{2} \left[\frac{z^4}{4} \right]_0^L \right. \\
 &\quad \left. - \frac{3}{2} qL^2 \left[\frac{z^2}{2} \right]_0^L + \frac{3}{2} qL \left[\frac{z^3}{3} \right]_0^L \right\} + \frac{X}{EI} \left\{ + \frac{1}{4} \left[\frac{z^3}{3} \right]_0^L + \frac{L^2}{4} \left[z \right]_0^L + \frac{1}{4} \left[\frac{z^3}{3} \right]_0^L + \frac{L}{2} \left[\frac{z^2}{2} \right]_0^L \right\}
 \end{aligned}$$

$$\begin{aligned}
& + L^2 \left[\frac{7}{2} \right]_0^L + \left[\frac{7}{3} \right]_0^L - 2L \left[\frac{7}{2} \right]_0^L \left\} - \frac{\alpha \Delta T}{h} \frac{1}{2} \left[\frac{7}{2} \right]_0^L = \\
& = \frac{1}{EI} \left\{ -\frac{1}{24} qL^4 - \frac{1}{8} qL^4 - \frac{1}{16} qL^4 + \frac{3}{16} qL^4 + \frac{1}{8} qL^4 + \frac{1}{2} qL^4 - \frac{1}{8} qL^4 - \frac{3}{4} qL^4 + \frac{1}{2} qL^4 \right\} + \\
& + \frac{X}{EI} \left\{ \frac{L^3}{12} + \frac{L^3}{4} + \frac{L^3}{12} + \frac{L^3}{4} + L^3 + \frac{L^3}{3} - L^3 \right\} - \frac{\alpha \Delta T}{h} \frac{L^2}{4} = \\
& = + \frac{10}{48} \frac{qL^4}{EI} + \frac{XL^3}{EI} - \frac{L^2}{4} \frac{\alpha \Delta T}{h} =
\end{aligned}$$

$\Delta_{ve} = \Delta_{vi}$ fornire

$$- \varepsilon X + \frac{1}{2} \eta_A^0 = \frac{10}{48} \frac{qL^4}{EI} + \frac{XL^3}{EI} - \frac{L^2}{4} \frac{\alpha \Delta T}{h}$$

$$- \frac{XL^3}{EI} + \frac{1}{2} \frac{qL^4}{EI} = \frac{10}{48} \frac{qL^4}{EI} + \frac{XL^3}{EI} - \frac{1}{4} \frac{qL^4}{EI}$$

$$2 \frac{XL^3}{EI} = \frac{qL^4}{EI} \left[-\frac{10}{48} + \frac{1}{4} + \frac{1}{2} \right]$$

$$X = + \frac{13}{48} \cdot \frac{1}{2} qL$$

$$X = + \frac{13}{48} qL \quad \text{Positivo, ok!}$$